Contoh 1

Ekspansi bentuk aljabar $(x-1)(x-2)(x-3)(x-4)$

$y = x^2 - 5x + 5$

$= (x-1)(x-4)\ (x-2)(x-3) = (x^2-5x+4)(x^2-5x+6)$

$= (y-1)(y+1) = y^2 - 1 = (x^2-5x+5)^2 - 1$

$= x^4 + 25x^2 + 25 - 10x^3 + 10x^2 - 50x - 1$

$= x^4 - 10x^3 + 35x^2 - 50x + 24$

Contoh 2

$(5xy - (3x^2 - 2))(5xy + (3x^2 - 2))$

Ekspansi bentuk aljabar $(5xy - 3x^2 + 2)(5xy + 3x^2 - 2)$

$= (5xy)^2 - (3x^2 - 2)^2$

$= 25x^2y^2 - (9x^4 - 12x^2 + 4)$

$= 25x^2y^2 - 9x^4 + 12x^2 - 4$

Contoh 3

$x^2 - 3x + 1 = 0$

$x + \frac{1}{x} - 3 = 0 \Rightarrow x + \frac{1}{x} = 3$

Jika $x^2 - 3x + 1 = 0$. Sederhanakan bentuk $\frac{x^4 - x^2 + 1}{x^4 + x^2 + 1}$.

$$\frac{x^4 - x^2 + 1}{x^4 + x^2 + 1} = \frac{x^2 - 1 + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}}$$

$$= \frac{7 - 1}{7 + 1} = \frac{6}{8} = \frac{3}{4}$$

$$x + \frac{1}{x} = 3$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 \cdot x \cdot \frac{1}{x} = 9$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 7$$

JELAJAH NALAR

Contoh 4

$a^2+b^2+2ab = (a+b)^2$

Tentukan semua bilangan asli n sehingga $2^{20} + 2^{26} + 2^n$ adalah bilangan kuadrat

a) $2^{26} + 2^{20} + 2^n = (2^{13})^2 + 2 \cdot 2^{13} \cdot 2^6 + 2^n$

$= (2^{13} + 2^6)^2 - 2^{12} + 2^n$

Solusi $n = 12$

a) $2^{20} + 2^{26} + 2^n = (2^{10})^2 + 2 \cdot 2^{10} \cdot 2^{15} + 2^n$

$= (2^{10} + 2^{15})^2 - 2^{30} + 2^n$

Solusi $n = 30$

c) $2^{26} + 2^{20} + 2^n = (2^{13})^2 + (2^{10})^2 + 2^n$

$= (2^{13} + 2^{10})^2 - 2 \cdot 2^{13} \cdot 2^{10} + 2^n$

$= (2^{13} + 2^{10})^2 - 2^{24} + 2^n$

Solusi $n = 24$

$Hp = \{12, 24, 30\}$

Contoh 5

$6x^3 - x^2 - 3x + 2026$

$= (6x^3 + 2x^2 - 2x) - (3x^2 - x - 1) + 2025$

$= 2x(3x^2 - x - 1) - (3x^2 - x - 1) + 2025 = 2025$

Jika $3x^2 + x = 1$, sederhanakan bentuk $6x^3 - x^2 - 3x + 2026$.

$3x^2 + x - 1$

$$
\begin{array}{r|l}
 & 2x - 1 \\
\hline
1 & 6x^3 - x^2 - 3x + 2026 \\
 & 6x^3 + 2x^2 - 2x \quad - \\
\hline
 & -3x^2 - x + 2025 \\
 & -3x^2 - x + 1 \quad - \\
\hline
 & 2025
\end{array}
$$

$= (2x-1)(3x^2 + x - 1) + 2025$

$= 2025$

JELAJAH NALAR

Contoh 6

$1 = P(2) = 2^{25}a + 2^{15}b + 2^5c + 26$

$2^{25}a + 2^{15}b + 2^5c = -25$

Jika $P(x) = ax^{25} + bx^{15} + cx^5 + 26$ dan $P(2) = 1$. Tentukan nilai dari $P(-2)$.

$P(-2) = a(-2)^{25} + b(-2)^{15} + c(-2)^5 + 26$

$= -\left[2^{25}a + 2^{15}b + 2^5c\right] + 26$

$= -(-25) + 26 = 51$

Contoh 7

$\star (a+b)(a+c)(b+c) = (a^2+ab+ac+bc)(b+c)$

$= a^2b + ab^2 + abc + b^2c + a^2c + abc + ac^2 + bc^2$

$= a^2b + ab^2 + a^2c + ac^2 + b^2c + bc^2 + 2abc$

Jika $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c}$. Tentukan nilai dari

$abc(a+b)(a+c)(b+c)(a+b+c) = 0$.

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = \frac{1}{a+b+c} \Leftrightarrow \frac{ab+ac+bc}{abc} = \frac{1}{a+b+c} \Leftrightarrow (a+b+c)(ab+ac+bc) = abc$

$= a^2b + a^2c + abc + ab^2 + abc + b^2c + abc + ac^2 + bc^2 = abc$.

$\Rightarrow a^2b + ab^2 + a^2c + ac^2 + b^2c + bc^2 + 2abc = 0$

$\Rightarrow (a+b)(b+c)(c+a) = 0$.

JELAJAH NALAR

Contoh 8

$x^4 + x^3 + x^2 + x = -1$

Jika $x^4 + x^3 + x^2 + x + 1 = 0$. Tentukan nilai dari $x^{2025} + x^{2030} + 1$.

$x(x^4 + x^3 + x^2 + x + 1) = x^5 + x^4 + x^3 + x^2 + x = x^5 - 1 = 0$

$\Rightarrow x^5 = 1$

$x^{2025} + x^{2030} + 1 = 3$

JELAJAH NALAR

Contoh 9

$a+b = \cdots$
$b+c = \cdots$
$c+a = \cdots$

Jika $ab = 6, bc = 15, ac = 10$. Tentukan nilai dari $a^2 + b^2 + c^2$.

$a^2b^2c^2 = ab \cdot bc \cdot ac = 6 \cdot 15 \cdot 10 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 2 \cdot 5$

$(abc)^2 = (2 \cdot 3 \cdot 5)^2 \Rightarrow abc = 30$

$a(bc) = a(15) = 30 \Rightarrow a = 2$

$b = 3$

$c = 5$

$a^2 + b^2 + c^2$
$= 2^2 + 3^2 + 5^2$
$= 38$

Contoh 10

Jika $a_{2026}x^{2026} + a_{2025}x^{2025} + \cdots + a_1 x + a_0 = (x^2 - x + 1)^{1013}$.
Tentukan nilai dari $a_0 + a_2 + \cdots + a_{2026}$.

$1 = (1^2 - 1 + 1)^{1013} = a_0 + a_1 + a_2 + a_3 + \cdots + a_{2025} + a_{2026}$

$3^{1013} = ((-1)^2 - (-1) + 1)^{1013} = a_0 - a_1 + a_2 - a_3 + \cdots - a_{2025} + a_{2026}$

$1 + 3^{1013} = 2(a_0 + a_2 + \cdots + a_{2026})$

$a_0 + a_2 + a_4 + \cdots + a_{2026} = \frac{1 + 3^{1013}}{2}$ //

JELAJAH NALAR