Contoh 1

Tentukan nilai dari $123456789 \times 999999999$

- Lebay
- Ga Mungkin hitung Manual

$123456789 \times (1.000.000.000 - 1)$

$= 123456789\,000000000 - 123456789$

$= 123456788876543211$

$$\begin{array}{r} 123456789\,000\,000\,000 \\ 123456789 \\ \hline 123456788876543211 \end{array}$

Contoh 2

Jika $x + \frac{1}{x} = 3$, tentukan nilai dari $x^3 + \frac{1}{x^3}$.

$(a+b)^3 = a^3 + b^3 + 3ab\,(a+b)$

$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x\right)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right)$

$3^3 = x^3 + \frac{1}{x^3} + 3\,(1)(3)$

$27 = x^3 + \frac{1}{x^3} + 9$

$x^3 + \frac{1}{x^3} = 27 - 9 = 18$

Contoh 3

Tentukan nilai dari $\dfrac{2027^2}{2026^2+2028^2-2}$

Anggap $x = 2027$

$$\frac{x^2}{(x-1)^2+(x+1)^2-2} = \frac{x^2}{x^2-2x+1+x^2+2x+1-2}$$

$$= \frac{x^2}{2x^2} = \frac{1}{2}$$

Contoh 4

Tentukan nilai dari $\frac{83^3+17^3}{83\times66+17^2}$

$$\frac{(83+17)(83^2-83\times17+17^2)}{83\times66+17^2} = \frac{100(83(83-17)+17^2)}{83\times66+17^2}$$

$$= \frac{100(83\times66+17^2)}{83\times66+17^2} = 100$$

Contoh 5

$$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$

$$\frac{1}{k} - \frac{1}{k+1} = \frac{k+1-k}{k(k+1)} = \frac{1}{k(k+1)}$$

Tentukan nilai dari

$$\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+\frac{1}{90}$$

$$\frac{1}{2\times3}+\frac{1}{3\times4}+\frac{1}{4\times5}+\frac{1}{5\times6}+\frac{1}{6\times7}+\frac{1}{7\times8}+\frac{1}{8\times9}+\frac{1}{9\times10}$$

$$= \frac{1}{2} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{6}} + \cancel{\frac{1}{6}} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{7}} - \cancel{\frac{1}{8}} + \cancel{\frac{1}{8}} - \cancel{\frac{1}{9}} + \cancel{\frac{1}{9}} - \frac{1}{10}$$

$$= \frac{1}{2} - \frac{1}{10} = \frac{5-1}{10} = \frac{4}{10} = \frac{2}{5}$$

Contoh 6

$$\frac{1}{k(k+m)} = \frac{1}{m}\left[\frac{1}{k} - \frac{1}{k+m}\right]$$

Tentukan nilai dari

$$\frac{1}{10} + \frac{1}{40} + \frac{1}{88} + \frac{1}{154} + \frac{1}{238}$$

$$\frac{1}{2\times5} + \frac{1}{5\times8} + \frac{1}{8\times11} + \frac{1}{11\times14} + \frac{1}{14\times17}$$

$$= \frac{1}{3}\left(\frac{1}{2} - \frac{1}{5}\right) + \frac{1}{3}\left(\frac{1}{5} - \frac{1}{8}\right) + \frac{1}{3}\left(\frac{1}{8} - \frac{1}{11}\right) + \frac{1}{3}\left(\frac{1}{11} - \frac{1}{14}\right) + \frac{1}{3}\left(\frac{1}{14} - \frac{1}{17}\right)$$

$$= \frac{1}{3}\left(\frac{1}{2} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{8}} + \cancel{\frac{1}{8}} - \cancel{\frac{1}{11}} + \cancel{\frac{1}{11}} - \cancel{\frac{1}{14}} + \cancel{\frac{1}{14}} - \frac{1}{17}\right) = \frac{1}{3}\left(\frac{1}{2} - \frac{1}{17}\right)$$

$$= \frac{1}{3}\left(\frac{15}{34}\right) = \frac{5}{34}$$

Contoh 7

$$\frac{1}{\sqrt{k}+\sqrt{k+1}} \cdot \frac{\sqrt{k}-\sqrt{k+1}}{\sqrt{k}-\sqrt{k+1}} = \frac{\sqrt{k}-\sqrt{k+1}}{k-(k+1)} = -\sqrt{k}+\sqrt{k+1}$$

Tentukan nilai dari

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{99}+\sqrt{100}}$$

$$= -\sqrt{1}+\sqrt{2}-\sqrt{2}+\sqrt{3}-\sqrt{3}+\sqrt{4}+\cdots-\sqrt{99}+\sqrt{100}$$

$$= -1+10 = 9$$

Contoh 8

$a^4 + 4b^4 = (a^2)^2 + (2b^2)^2 = (a^2+2b^2)^2 - 2(a^2)(2b^2)$
$= (a^2+2b^2)^2 - 4a^2b^2 = (a^2+2b^2-2ab)(a^2+2b^2+2ab)$

Tentukan semua bilangan asli n sehingga $n^4 + 4$ adalah bilangan prima

$n^4 + 4 = (n^2-2n+2)(n^2+2n+2)$

Agar n^4+4 prima, maka $n^2-2n+2 = 1$

$\Leftrightarrow n^2-2n+1=0 \Leftrightarrow (n-1)^2 = 0 \Leftrightarrow n = 1$

$n = 1 \Rightarrow 1^4+4 = 5$

Contoh 9

$1 = 2 - 1$

Tentukan nilai dari $(2 + 1)(2^2 + 1)(2^{2^2} + 1) \ldots (2^{2^n} + 1)$

$= (2-1)(2+1) \ldots (2^{2^n}+1)$

$= (2^2-1)(2^2+1) \ldots (2^{2^n}+1) = ((2^2)^2-1)(2^{2^2}+1) \ldots (2^{2^n}+1)$

$= (2^{2^2}-1)(2^{2^2}+1) \ldots (2^{2^n}+1) = \ldots$

$= (2^{2^n}-1)(2^{2^n}+1) = (2^{2^n})^2 - 1$

$= 2^{n+1} - 1$

Contoh 10

$1 = 2 - 1 = (\sqrt[3]{2})^3 - 1$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Tentukan nilai dari

$$\left(1 + \frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1}\right)^3$$

$1 = (\sqrt[3]{2})^3 - 1 = (\sqrt[3]{2} - 1)((\sqrt[3]{2})^2 + (\sqrt[3]{2}) + 1)$

$= (\sqrt[3]{2} - 1)(\sqrt[3]{4} + \sqrt[3]{2} + 1) \Leftrightarrow \sqrt[3]{2} - 1 = \frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1}$

$\Leftrightarrow \sqrt[3]{2} = 1 + \frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} \Leftrightarrow 2 = \left(1 + \frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1}\right)^3$